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The Jacobi polynomials QCD analysis for the polarized structure function

S. Atashbar Tehrani[ac] **and Ali N. Khorramian**[bc]

[a] *Physics Department, Persian Gulf University, P.O.Box 75169-13798, Boushehr, Iran*
[b] *Physics Department, Semnan University, P.O.Box 35195-363, Semnan, Iran*
[c] *Institute for Studies in Theoretical Physics and Mathematics (IPM), P.O.Box 19395-5531, Tehran, Iran*
E-mail: atashbar@ipm.ir, khorramiana@theory.ipm.ac.ir

Abstract: We present the results of our QCD analysis for polarized quark distribution and structure function $xg_1(x, Q^2)$. We use very recently experimental data to parameterize our model. New parameterizations are derived for the quark and gluon distributions for the kinematic range $x \in [10^{-8}, 1]$, $Q^2 \in [1, 10^6]\,\mathrm{GeV}^2$. The analysis is based on the Jacobi polynomials expansion of the polarized structure functions. Our calculations for polarized parton distribution functions based on the Jacobi polynomials method are in good agreement with the other theoretical models. The values of Λ_{QCD} and $\alpha_s(M_z)$ are determined.

Keywords: Phenomenological Models, NLO Computations, Spin and Polarization Effects, Deep Inelastic Scattering.

Contents

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1. Introduction

The nature of the short-distance structure of polarized nucleons is one of the central questions of present day hadron physics. For more than sixteen years, polarized inclusive deep inelastic scattering has been the main source of information on how the individual partons in the nucleon are polarized at very short distances.

The theoretical and experimental status on the spin structure of the nucleon has been discussed in great detail in several recent reviews (see, e.g., refs. [1 – 4]. During the recent years several comprehensive analysis of the polarized deep inelastic scattering (DIS) data, based on next-to-leading-order quantum chromodynamics, have appeared in refs.[5-29]. In these analysis the polarized parton density functions (PPDFs) are either written in terms of the well-known parameterizations of the unpolarized PDFs or parameterized independently, and the unknown parameters are determined by fitting the polarized DIS data.

Determination of parton distributions in a nucleon in the framework of quantum chromodynamics (QCD) always involves some model-dependent procedure. Instead of relying on mathematical simplicity as a guide, we take a viewpoint in which the physical picture of the nucleon structure is emphasized. That is, we consider the model for the nucleon which is compatible with the description of the bound state problem in terms of three constituent quarks. We adopt the view that these constituent quarks in the scattering problems should be regarded as the valence quark clusters rather than point-like objects. They have been referred to as *valons*. In the valon model, the proton consists of two "up" and one "down" valons. These valons thus, carry the quantum numbers of the respective valence quarks. Hwa and et al. [30-38] found evidence for the valons in the deep inelastic neutrino scattering data, suggested their existence and applied it to a variety of phenomena. Hwa [39] has also successfully formulated a treatment of the low-p_T reactions based

on a structural analysis of the valons. In [40] Hwa and Yang refined the idea of the valon model and extracted new results for the valon distributions. In [41 – 43] unpolarized PDFs and hadronic structure functions in the NLO approximation were extracted.

In ref. [25] the polarized valon model is applied to determine the quark helicity distributions and polarized proton structure functions in the NLO approximation by using the Bernstein polynomial approach.

The extraction of the quark helicity distributions is one of the main tasks of the semi-inclusive deep inelastic scattering (SIDIS) experiments (HERMES [44], COMPASS [45], SMC [46]) with the polarized beam and target. Recently in ref. [28] the polarized valon model was applied and analyzed the flavor-broken light sea quark helicity distributions with the help of a Pauli-blocking ansatz. The results reported of this paper is good agreement with the HERMES experimental data for the quark helicity distributions in the nucleon for up, down, and strange quarks from semi-inclusive deep-inelastic scattering [44]. The reported results of ref. [44] is based on the Bernstein polynomial expansion method in the polarized valon model framework.

Since very recently experimental data are available from the HERMES collaboration [47] for the spin structure function g_1, therefore there is enough motivation to study and utilize the spin structure and quark helicity distributions extracted via the phenomenological model. Since these recently experimental data are just for different value of Q^2 and not fixed Q^2, one can not use the the Bernstein polynomial expansion method. Because in this method we need to use experimental data for each bin of Q^2 separately. So it seems suitable, to use the Jacobi polynomial expansion method.

In this paper we use the idea of the polarized valon model to obtain the PPDFs in the LO and NLO approximations. The results of the present analysis is based on the Jacobi polynomials expansion of the polarized structure function.

The plan of the paper is to give an introduction to the polarized valon distributions in section 2. Parametrization of parton densities are written down in this section. In section 3 we present a brief review of the theoretical background of the QCD analysis in two loops. The method of the QCD analysis of polarized structure function, based on Jacobi polynomials are written down in section 4. A description of the procedure of the QCD fit of g_1 data are illustrated in section 5. section 6 contains a results and conclusions of the QCD analysis.

2. Polarized valon distributions

The idea of nucleon as a bound state of three quarks was presented for the first time in ref. [48]. On the other hand the similar idea, which is called valon model was presented in ref. [37]. According to the valon model framework, one can assume a simple form for the exclusive valon distribution which facilitate the phenomenological analysis as follows

$$G_{UUD/p}(y_1, y_2, y_3) = g(y_1 y_2)^a y_3^b \delta(y_1 + y_2 + y_3 - 1) \,, \tag{2.1}$$

where y_i is the momentum fraction of the i'th valon. The U and D type inclusive valon distributions can be obtained by double integration over the unspecified variables

$$G_{U/p}(y) = \int dy_2 \int dy_3 G_{UUD/p}(y, y_2, y_3) = gB(a+1, b+1)y^a(1-y)^{a+b+1} , \quad (2.2)$$

$$G_{D/p}(y) = \int dy_1 \int dy_2 G_{UUD/p}(y_1, y_2, y) = gB(a+1, a+1)y^b(1-y)^{2a+1} . \quad (2.3)$$

The normalization parameter g has been fixed by

$$\int_0^1 G_{U/p}(y)dy = \int_0^1 G_{D/p}(y)dy = 1 , \quad (2.4)$$

and is equal to $g = [B(a+1, b+1)B(a+1, a+b+2)]^{-1}$, where $B(m, n)$ is the Euler-Beta function. R.C. Hwa and C. B. Yang [40] have recalculated the unpolarized valon distribution in the proton with a new set of parameters. The new values of a , b are found to be $a = 1.76$ and $b = 1.05$.

To describe the quark distribution $q(x)$ in the valon model, one can try to relate the polarized quark distribution functions $q^\uparrow$ or $q^\downarrow$ to the corresponding valon distributions $G^\uparrow$ and $G^\downarrow$. The polarized valon can still have the valence and sea quarks that are polarized in various directions, so long as the net polarization is that of the valon. When we have only one distribution $q(x, Q^2)$ to analyze, it is sensible to use the convolution in the valon model to describe the proton structure in terms of the valons. In the case that we have two quantities, unpolarized and polarized distributions, there is a choice of which linear combination exhibits more physical contents. Therefore, in our calculations we assume a linear combination of $G^\uparrow$ and $G^\downarrow$ to determine respectively the unpolarized (G) and polarized (ΔG) valon distributions .

According to unpolarized and polarized valon model framework we have [25]

$$q_{i/p}(x, Q^2) = \sum_j \int_x^1 q_{i/j}\left(\frac{x}{y}, Q^2\right) G_{j/p}(y)\frac{dy}{y} , \quad (2.5)$$

and

$$\Delta q_{i/p}(x, Q^2) = \sum_j \int_x^1 \Delta q_{i/j}\left(\frac{x}{y}, Q^2\right) \Delta G_{j/p}(y)\frac{dy}{y} . \quad (2.6)$$

where $q_i \equiv q_i^\uparrow + q_i^\downarrow$ and $\Delta q_i \equiv q_i^\uparrow - q_i^\downarrow$ and also $G_j \equiv G_j^\uparrow + G_j^\downarrow$ and $\Delta G_j \equiv G_j^\uparrow - G_j^\downarrow$. As we can similarly see for the unpolarized case, the polarized quark distribution can be related to a polarized valon distribution in eq. (2.6).

The polarized parton distribution of the polarized constituent quarks in the eq. (2.6) is dependent on a probe [25]. It is assumed in more detail that the polarized constituent quarks distribution, that is, $\Delta q_{i/j}(z = \frac{x}{y}, Q^2)$ in the scale of Q_0^2 is equal to $\Delta q_{i/j}(z, Q_0^2) = \delta(1-z)$. From eq. (2.6) now is obvious that the polarized quark distribution in the scale of Q_0^2 is equal to polarized valon distribution.

Here we want to use the polarized valon distributions which introduced in [25]. According to improve polarized valon picture the nonsinglet polarized valon distribution functions are as following

$$\Delta G_j(y) = \Delta \mathcal{W}_j(y) \times G_j(y) \;, \tag{2.7}$$

where

$$\Delta \mathcal{W}_j(y) = \xi_j A_j y^{\alpha_j} (1-y)^{\beta_j} (1 + \gamma_j y + \eta_j y^{0.5}) \;, \tag{2.8}$$

the subscript j refers to U and D-valons. The motivation for choosing this functional form is that the y^{α_j} term controls the low-y behavior valon densities, and $(1-y)^{\beta_j}$ terms the large-y values. The remaining polynomial factor accounts for the additional medium-y values. The normalization constants A_j for U and D-valons are as following

$$\begin{aligned} {A_{_U}}^{-1} = [&B(a+\alpha_{_U}, 2+a+b+\beta_{_U}) + \eta_{_U} B(0.5+a+\alpha_{_U}, 2+a+b+\beta_{_U}) \\ &+\gamma_{_U} B(1+a+\alpha_{_U}, 2+a+b+\beta_{_U})] \, / B(1+a, 2+a+b) \;, \end{aligned} \tag{2.9}$$

and

$$\begin{aligned} {A_{_D}}^{-1} = [&B(b+\alpha_{_D}, 2+2a+\beta_{_D}) + \eta_{_D} B(0.5+b+\alpha_{_D}, 2+2a+\beta_{_D}) \\ &+\gamma_{_D} B(1+b+\alpha_{_D}, 2+2a+\beta_{_D})] \, / B(b+1, 2a+2) \;. \end{aligned} \tag{2.10}$$

These quantities are chosen such that the ξ_j are the first moments of $\Delta G_j(y)$, $\xi_j = \int_0^1 dy \Delta G_j(y)$. Here $B(a,b)$ is the Euler Beta-function being related to the Γ-function.

In the present approach the QCD-evolution equations are solved in MELLIN–N space as will describe in section 3. The MELLIN–transform of the valon densities is performed and MELLIN–N moments are calculated for complex arguments N by $\int_0^1 y^{N-1} \Delta G_{j/p}(y) dy$.

As seen from eq. (2.7) there are five parameters for each valon distribution. To meet both the quality of the present data and the reliability of the fitting program, the number of parameters has to be reduced. Assuming SU(3) flavor symmetry and a flavor symmetric sea one only has to derive one general polarized sea-quark distribution. The first moments of the polarized valon distributions $\Delta G_{_U}$ and $\Delta G_{_D}$ can be fixed by the SU(3) parameters F and D as measured in neutron and hyperon β-decays according to the relations :

$$\begin{aligned} 2\xi_{_U} - \xi_{_D} &= F + D \,, \\ 2\xi_{_U} + \xi_{_D} &= 3F - D \;. \end{aligned} \tag{2.11}$$

The factor 2 in eq. (2.11) is due to the existence of two-U type valons. A re-evaluation of F and D was performed in ref. [24] on the basis of updated β-decay constants [49] leading to

$$\begin{aligned} 2\xi_{_U} &= \;\; 0.926 \pm 0.014 \,, \\ \xi_{_D} &= -0.341 \pm 0.018 \;. \end{aligned} \tag{2.12}$$

This choice reduces the number of parameters to be fitted for each nonsinglet valon density to four.

On the other hand the singlet polarized valon distribution which is defined in [25] is as following

$$\Delta G_j^{'}(y) = \Delta \mathcal{W}_j^{'}(y) \times G_j(y) \tag{2.13}$$

where $\Delta \mathcal{W}_j^{'}(y)$ in eq. (2.13) has the following form

$$\Delta \mathcal{W}_j^{'}(y) = \Delta \mathcal{W}_j(y) \times \sum_{m=0}^{5} \mathcal{A}_m y^{\frac{m-1}{2}} \ . \tag{2.14}$$

The additional term in the above equation, ($\sum$ term), serves to control the behavior of the singlet sector at very low-y values and the $\Delta \mathcal{W}_j(y)$ defined in eq. (2.8).

In the next sections we need to use the MELLIN moments of eqs. (2.7), (2.13) to determine MELLIN moments of PDFs in our analysis.

3. The theoretical background of the QCD analysis

Let us define the MELLIN moments for the polarized structure function $g_1^p(x, Q^2)$:

$$g_1^p(N, Q^2) = \int_0^1 \ x^{N-1} g_1^p(x, Q^2) dx \tag{3.1}$$

In the QCD-improved quark parton model (QPM), i.e., at leading twist, and to leading logarithmic order in the running strong coupling constant $\alpha_s(Q^2)$ of Quantum-Chromodynamics (LO QCD), the deep-inelastic scattering off the nucleon can be interpreted as the incoherent superposition of virtual-photon interactions with quarks of any flavor q. By angular momentum conservation, a spin-$\frac{1}{2}$ parton can absorb a hard photon only when their spin orientations are opposite. The spin structure function g_1 has then a probabilistic interpretation, which for the proton reads [50]

$$\begin{aligned} g_1^p(N, Q^2) &= \frac{1}{2} \sum_q e_q^2 \left[\Delta q(N, Q^2) + \Delta \bar{q}(N, Q^2) \right] \\ &= \frac{1}{2} \langle e^2 \rangle \left[\Delta q_S(N, Q^2) + \Delta q_{\rm NS}(N, Q^2) \right] . \end{aligned} \tag{3.2}$$

Here, the quantity $-Q^2$ is the squared four-momentum transferred by the virtual photon, N is the order of moments, e_q is the charge, in units of the elementary charge $|e|$, of quarks of flavor q, $\langle e^2 \rangle = \sum_q e_q^2 / N_q$ is the average squared charge of the N_q active quark flavors, and $\Delta q(N, Q^2)$ is the quark helicity distribution for quarks of flavor q. Correspondingly, $\Delta \bar{q}(N, Q^2)$ is anti-quark helicity distributions. Moreover the flavor singlet and flavor non-singlet quark helicity distributions are defined as

$$\begin{aligned} \Delta q_S(N, Q^2) &= \sum_q \left[\Delta q(N, Q^2) + \Delta \bar{q}(N, Q^2) \right] \\ &\equiv \Delta \Sigma(N, Q^2), \end{aligned} \tag{3.3}$$

and

$$\Delta q_{\rm NS}(N, Q^2) = \frac{1}{\langle e^2 \rangle} \sum_q e_q^2 \left[\Delta q(N, Q^2) + \Delta \bar{q}(N, Q^2) \right] - \Delta q_S(N, Q^2). \tag{3.4}$$

For the analysis presented in this paper, only the three lightest quark flavors, $q = u, d, s$, are taken into account and the number of active quark flavors N_q is equal to three.

The twist-2 contributions to the structure function $g_1(N, Q^2)$ can be represented in terms of the polarized parton densities and the coefficient functions ΔC_i^N in the MELLIN -N space by [2]

$$g_1^p(N,Q^2) = \frac{1}{2}\sum_q e_q^2 \left\{ \left(1 + \frac{\alpha_s}{2\pi}\Delta C_q^N\right)[\Delta q(N,Q^2) + \Delta\bar{q}(N,Q^2)] + \frac{\alpha_s}{2\pi} 2\Delta C_g^N \Delta g(N,Q^2) \right\}, \tag{3.5}$$

in this equation the NLO running coupling constant is given by

$$\alpha_s(Q^2) \cong \frac{1}{b\log\frac{Q^2}{\Lambda^2_{\overline{MS}}}} - \frac{b'}{b^3}\frac{\ln\left(\ln\frac{Q^2}{\Lambda^2_{\overline{MS}}}\right)}{\left(\ln\frac{Q^2}{\Lambda^2_{\overline{MS}}}\right)^2}, \tag{3.6}$$

where $b = \frac{33-2f}{12\pi}$ and $b' = \frac{153-19f}{24\pi^2}$. In the above equations, we choose $Q_0 = 1\ GeV^2$ as a fixed parameter and Λ is an unknown parameter which can be obtained by fitting to experimental data.

In eq. (3.5), $\Delta q(N,Q^2) = \Delta q_v(N,Q^2) + \Delta\bar{q}(N,Q^2)$, $\Delta\bar{q}(N,Q^2)$ and $\Delta g(N,Q^2)$ are moments of the polarized parton distributions in a proton. ΔC_q^N, ΔC_g^N are also the N-th moments of spin-dependent Wilson coefficients given by

$$\Delta C_q^N = \frac{4}{3}\left[-S_2(N) + (S_1(N))^2 + \left(\frac{3}{2} - \frac{1}{N(N+1)}\right)S_1(N) + \frac{1}{N^2} + \frac{1}{2N} + \frac{1}{N+1} - \frac{9}{2}\right], \tag{3.7}$$

and

$$\Delta C_g^N = \frac{1}{2}\left[-\frac{N-1}{N(N+1)}(S_1(N)+1) - \frac{1}{N^2} + \frac{2}{N(N+1)}\right], \tag{3.8}$$

with $S_k(N)$ defined as in ref. [2].

According to improved polarized valon model framework, determination of the moments of parton distributions in a proton can be done strictly through the moments of the polarized valon distributions.

The moments of PPDFs are denoted respectively by: $\Delta u_v(N,Q^2)$, $\Delta d_v(N,Q^2)$, $\Delta\Sigma(N,Q^2)$ and $\Delta g(N,Q^2)$. Therefore, the moments of the polarized u and d-valence quark in a proton are convolutions of two moments:

$$\Delta u_v(N,Q^2) = 2\Delta M^{\rm NS}(N,Q^2) \times \Delta M'_{U/p}(N), \tag{3.9}$$

$$\Delta d_v(N,Q^2) = \Delta M^{\rm NS}(N,Q^2) \times \Delta M'_{D/p}(N), \tag{3.10}$$

In the above equation $M'_{j/p}(N)$ is the moment of $\Delta G_{j/p}(y)$ distribution, i.e. $\Delta M'_{j/p}(N) = \int_0^1 y^{N-1}\Delta G_{j/p}(y)dy$.

The moment of the polarized singlet distribution (Σ) is as follows:

$$\Delta\Sigma(N,Q^2) = \Delta M^S(N,Q^2)(2\Delta M''_{U/p}(N) + \Delta M''_{D/p}(N)). \tag{3.11}$$

here $M''_{j/p}(N)$ is the moment of $\Delta G'_{j/p}(y)$ distribution, i.e. $\Delta M''_{j/p}(N) = \int_0^1 y^{N-1}\Delta \cdot$ $\cdot G'_{j/p}(y)dy$. For the gluon distribution we have

$$\Delta g(N,Q^2) = \Delta M^{gq}(N,Q^2)(2\Delta M^{'}_{U/p}(N) + \Delta M^{'}_{D/p}(N)) \ , \tag{3.12}$$

where $\Delta M^{\rm NS}(N,Q^2)$ in eqs. (3.9), (3.10), $\Delta M^{S}(N,Q^2)$ in eq. (3.11) and also $\Delta M^{gq}(N,Q^2)$ which is the quark-to-gluon evolution function, are given in ref. [25].

It is obvious that the final form for $g_1(N,Q^2)$ involves the total of 15 unknown parameters. If the parameters can be obtained then the computation of all moments of the PPDFs and the polarized structure function (PSF), $g_1(N,Q^2)$, are possible.

4. The method of the QCD analysis of PSF

The evolution equations allow one to calculate the Q^2-dependence of the PPD's provided at a certain reference point Q_0^2. These distributions are usually parameterized on the basis of plausible theoretical assumptions concerning their behavior near the end points $x = 0, 1$.

In the phenomenological investigations of the polarized and unpolarized structure functions, for example xg_1^p or xF_3^p for a given value of Q^2, only a limited number of experimental points, covering a partial range of values of x, are available. Therefore, one cannot directly determine the moments. A method devised to deal with this situation is to take averages of the structure function weighted by suitable polynomials.

The evolution equation can be solved and QCD predictions for PSF obtained with the help of various methods. For example we can use the Bernstein polynomial to determine PPD's in the NLO approximation to obtain some unknown parameters to parameterize PPD's at Q_0^2. In this way, we can compare theoretical predictions with the experimental results for the Bernstein averages just in MELLIN-N space. To obtain these experimental averages from the E143 and SMC data [51, 52], we need to fit $xg_1(x,Q^2)$ for each bin in Q^2 separately [25].

If we want to take into account very recent experimental data [47] for all range value of Q^2 to determine PPDFs, it is convenience to apply Jacobi polynomials expansion and not the Bernstein one. The advantage of application for all data points, especially very recently HERMES experimental data [47], and not just a series of data points, is our motivation to use Jacobi polynomials and not Bernstein polynomial to study spin dependent of parton distribution function.

One of the simplest and fastest possibilities in the PSF reconstruction from the QCD predictions for its Mellin moments is Jacobi polynomials expansion. The Jacobi polynomials are especially suited for this purpose since they allow one to factor out an essential part of the x-dependence of the SF into the weight function [53]. Thus, given the Jacobi moments $a_n(Q^2)$, a structure function $f(x,Q^2)$ may be reconstructed in a form of the series [54]–[58]

$$xf(x,Q^2) = x^\beta(1-x)^\alpha \sum_{n=0}^{N_{\rm max}} a_n(Q^2)\Theta_n^{\alpha,\beta}(x), \tag{4.1}$$

where $N_{\max}$ is the number of polynomials and $\Theta_n^{\alpha,\beta}(x)$ are the Jacobi polynomials of order n,

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) x^j, \tag{4.2}$$

where $c_j^{(n)}(\alpha,\beta)$ are the coefficients that expressed through Γ-functions and satisfy the orthogonality relation with the weight $x^\beta(1-x)^\alpha$ as following

$$\int_0^1 dx\; x^\beta (1-x)^\alpha \Theta_k^{\alpha,\beta}(x)\Theta_l^{\alpha,\beta}(x) = \delta_{k,l}\,, \tag{4.3}$$

For the moments, we note that the Q^2 dependence is entirely contained in the Jacobi moments

$$\begin{aligned} a_n(Q^2) &= \int_0^1 dx\; x f(x,Q^2)\Theta_k^{\alpha,\beta}(x) \\ &= \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) f(j+2,Q^2)\;. \end{aligned} \tag{4.4}$$

obtained by inverting eq. 4.1, using eqs. (4.2), (4.3) and also definition of moments, $f(j,Q^2) = \int_0^1 dx\; x^{j-2} x f(x,Q^2)$.

Using eqs. (4.1), (4.4) now, one can relate the PSF with its Mellin moments

$$x g_1^{N_{\max}}(x,Q^2) = x^\beta (1-x)^\alpha \sum_{n=0}^{N_{\max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) g_1(j+2,Q^2), \tag{4.5}$$

where $g_1(j+2,Q^2)$ are the moments determined by eqs. (3.2), (3.5). $N_{\max}$, α and β have to be chosen so as to achieve the fastest convergence of the series on the r.h.s. of eq. (4.5) and to reconstruct xg_1 with the required accuracy. In our analysis we use $N_{\max} = 9$, $\alpha = 3.0$ and $\beta = 0.5$. The same method has been applied to calculate the nonsinglet structure function xF_3 from their moments [59-63].

Obviously the Q^2-dependence of the polarized structure function is defined by the Q^2-dependence of the moments.

5. The procedure of the QCD fit of g_1^p data

The remarkable growth of experimental data on inclusive polarized deep inelastic scattering of leptons off nucleons over the last years allows to perform refined QCD analyzes of polarized structure functions in order to reveal the spin-dependent partonic structure of the nucleon. For the QCD analysis presented in the present paper the following data sets are used: the HERMES proton data [64, 47], the SMC proton data [65], the E143 proton data [51], the EMC proton data [66, 67]. The number of the published data points above $Q^2 = 1.0$ GeV^2 for the different data sets are summarized in table 1 for data on g_1 together with the $x-$ and Q^2-ranges for different experiments.

In the fitting procedure we started with the 15 parameters selected, i.e. 4 parameters for each non-singlet polarized valon distribution, 6 parameters for singlet polarized

Experiment	x-range	Q^2-range [GeV^2]	number of data points	ref.
E143(p)	0.031 – 0.749	1.27 – 9.52	28	[51]
E143(p)	0.031 – 0.749	2, 3, 5 (Fixed)	84	[51]
HERMES(p)	0.028 – 0.660	1.01 – 7.36	19	[64]
HERMES(p)	0.023 – 0.660	2.5 (Fixed)	20	[64]
HERMES(p)	0.026 – 0.731	1.12-14.29	62	[47]
SMC(p)	0.005 – 0.480	1.30 – 58.0	12	[65]
SMC(p)	0.005 – 0.480	10 (Fixed)	12	[65]
EMC(p)	0.015 – 0.466	3.50 – 29.5	10	[66]
EMC(p)	0.015 – 0.466	3.50 – 29.5	10	[67]
proton			257	

Table 1: Published data points above $Q^2 = 1.0\,\mathrm{GeV}^2$.

valon distribution and Λ_{QCD} to be determined. For this set of parameters the sea-quark distribution was assumed to be described according to SU(3) flavor symmetry.

In the further procedure we fixed 2 parameters at their values obtained in the first minimization and chose the first moment of polarized Non-singlet valon distributions in the polarized case [25]. The lack of constraining power of the present data on the polarized parton densities has to be stressed, however. Since only more precise data can improve the situation, so we add the recent experimental data achieved from HERMES group [47].

The final minimization was carried out under the above conditions and determined the remaining 15 parameters. The values and errors of these parameters along with those parameters fixed in the parametrization, section 2, are summarized in LO and NLO in table 2. The results on Λ_{QCD} are discussed separately in section 6. The starting scale of the evolution was chosen as $Q_0^2 = 1\,\mathrm{GeV}^2$.

Using the CERN subroutine MINUIT [68], we defined a global χ^2 for all the experimental data points and found an acceptable fit with minimum $\chi^2/\mathrm{d.o.f.} = 236.577/242 = 0.978$ in the LO case and $\chi^2/\mathrm{d.o.f.} = 225.920/242 = 0.933$ in the NLO case. In this table we compare the results reported in [25] which are based on Bernstein approach and the results of the present analysis. Also we obtain the uncertainties of the parameters in Jacobi Approach which are not calculated in Bernstein approach. We should notice that the Jacobi and Bernstein polynomials are merely used as a tool in the fitting procedure and the results are independent of it.

6. Results and conclusions of the QCD analysis

We have performed a QCD analysis of the inclusive polarized deep-inelastic charged lepton-nucleon scattering data to next-to-leading order and derived parameterizations of polarized valon distributions at a starting scale Q_0^2 together with the QCD-scale Λ_{QCD} in the polarized valon model framework.

	Bernstein Approach [25]		Jacobi Approach	
	LO	NLO	LO	NLO
	value	value	value	value
$\Lambda^{(4)}_{\rm QCD}$;MeV	203	235	201±110	245±58
$N_{_U}(=\xi_{_U} A_{_U})$	0.0020	0.0038	0.0015(fixed)	0.0018(fixed)
$\alpha_{_U}$	-2.3789	-2.1501	-2.4489±0.024	-2.3590±0.028
$\beta_{_U}$	-1.7518	-0.8859	-1.9050±0.196	-0.9683± 0.206
$\gamma_{_U}$	11.0804	10.6537	13.5420±0.378	22.6095± 0.418
$\eta_{_U}$	-1.4629	-0.1548	-1.8339±0.120	-1.9778± 0.150
$N_{_D}(=\xi_{_D} A_{_D})$	-0.005	-0.0046	-0.0029(fixed)	-0.0026(fixed)
$\alpha_{_D}$	-1.5465	-1.5859	-1.6059±0.029	-1.6638±0.034
$\beta_{_D}$	-1.8776	-1.5835	-2.2009±0.353	-1.6214± 0.453
$\gamma_{_D}$	8.5042	9.6205	10.2371±0.829	13.1966± 0.942
$\eta_{_D}$	-0.8608	-0.8410	0.8751±0.091	0.8483±0.142
$\mathcal{A}_0$	0.0004	-0.0025	0.0003± 10^{-5}	0.0052±10^{-5}
$\mathcal{A}_1$	0.2954	-3.1148	0.2611±0.191	-3.0034±0.241
$\mathcal{A}_2$	-6.9134	15.8114	-6.5429±0.174	14.8696± 0.214
$\mathcal{A}_3$	30.9851	-21.1500	29.6999±0.475	-18.8057± 0.051
$\mathcal{A}_4$	-39.7383	10.5025	-37.9379±0.202	8.4598± 0.298
$\mathcal{A}_5$	16.4605	-0.9162	15.5879±0.041	-0.3410±0.050
χ^2/ndf	154.98/123=1.260	115.62/123=0.940	236.577/242=0.978	225.920/242=0.933

Table 2: Parameter values in LO and NLO of the parameter fit for Bernstein and Jacobi approaches.

The analysis was performed using the Jacobi polynomials-method to determine the parameters of the problem in a fit to the data.

A new aspect in comparison with previous analyzes is that we determine the parton densities and the QCD scale in leading and next-to-leading order by using Jacobi polynomial expansion method.

Detailed comparisons were performed to the results obtained in other recent parameterizations [24, 23, 20, 27, 26]. The previous results are widely compatible with the present parameterizations. These distributions can be used in the numerical calculations for polarized high-energy scattering processes at hadron– and *ep*-colliders.

Looking at the Q^2 dependence of the structure function $g_1(x, Q^2)$ in intervals of x gives insight to the scaling violations in the spin sector. As in the unpolarized case the presence of scaling violations are expected to manifest in a slope changing with x. The proton data on $g_1(x, Q^2)$ have been plotted in such a way in figure 1 and confronted with the QCD NLO curves of the present analysis. Corresponding curves of the parameterizations [24, 23, 20, 27, 26] are also shown. Slight but non-significant differences between the different analyzes are observed in the intervals at low values of x. However, the data are well covered within the errors by all analyzes.

In figures 2–3 the fitted parton distribution functions, in leading and next-to-leading

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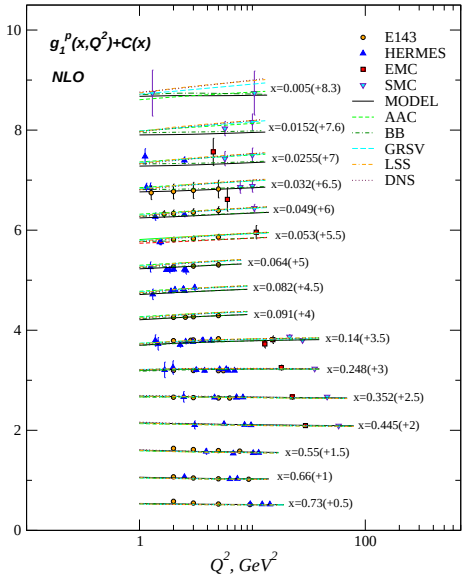

Figure 1: The polarized structure function g_1^p as function of Q^2 in intervals of x. The error bars shown are the statistical and systematic uncertainties added in quadrature. The data are well described by our QCD NLO curves (solid lines) which based on the valon model and Jacobi polynomials expansion method. The values of $C(x)$ are given in parentheses. Also shown are the QCD NLO curves obtained by AAC (dashed lines) [24], BB (dashed-dotted lines) [23], GRSV (long-dashed lines) [20], LSS (dashed-dashed-dotted lines) [27] and DNS (dotted lines) [26] for comparison.

order for all sets of parameterizations [24, 23, 20, 27] and their errors are presented at the starting scale Q_0^2.

The polarized structure function $xg_1^p(x.Q^2)$ measured in the interval $3.0\,\mathrm{GeV}^2 < Q^2 < 5.0\,\mathrm{GeV}^2$, figure 4, using the world asymmetry data is well described by our QCD NLO curve. We also compare to corresponding representations of the parameterizations [27, 20, 23, 24], which are compatible within the present results.

In figures 5–8 the scaling violations of the individual polarized momentum densities are depicted in the range $x\,\epsilon\,[10^{-3},1]$, $Q^2\,\epsilon\,[1,10^4]$ GeV choosing the NLO distributions. The up-valence distribution $x\Delta u_v$, figure 5, evolves towards smaller values of x and the peak around $x \sim 0.25$ becomes more flat in the evolution from $Q^2 = 1\,\mathrm{GeV}^2$ to $Q^2 = 10^4\,\mathrm{GeV}^2$. Statistically this distribution is constrained best among all others. The down-valence distribution $x\Delta d_v$, figure 6, remains negative in the same range, although it is less constraint by the present data than the up-valence density. Also here the evolution is towards smaller values of x and structures at larger x flatten out.

The momentum density of the polarized singlet quark $x\Delta\Sigma$, figure 7, is positive in the kinematic range shown for all Q^2 and for $x \geq 0.1$, but changes sign for lower values of x. The maximum of the distribution at $Q^2 = 1\,\mathrm{GeV}^2$ around $x \sim 0.3$ moves to $x \sim 0.2$ at $Q^2 = 10^4\,\mathrm{GeV}$. At the same time a minimum around $x \sim 0.03$ moves to $x \sim 0.002$.

The momentum density of the polarized gluon $x\Delta G$, figure 8, is positive in the kine-

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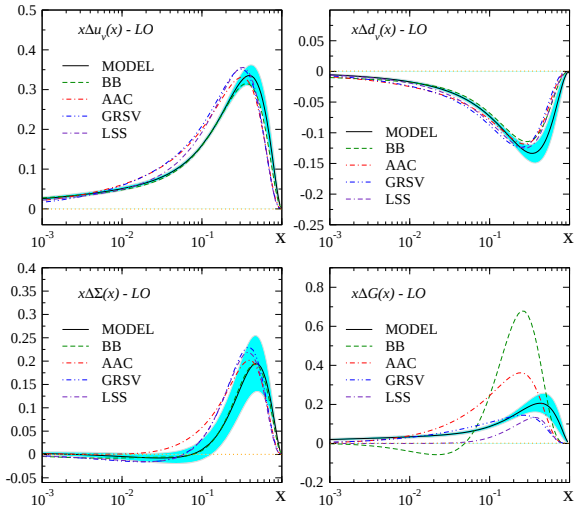

Figure 2: LO polarized parton distributions at the input scale Q_0^2=1.0 GeV^2 compared to results obtained by BB model (dashed line) (ISET=1) [23], AAC (dashed-dotted line) (ISET=1) [24], GRSV (dashed-dotted dotted line) (ISET=3) [20] and LSS (dashed-dashed dotted line) (ISET

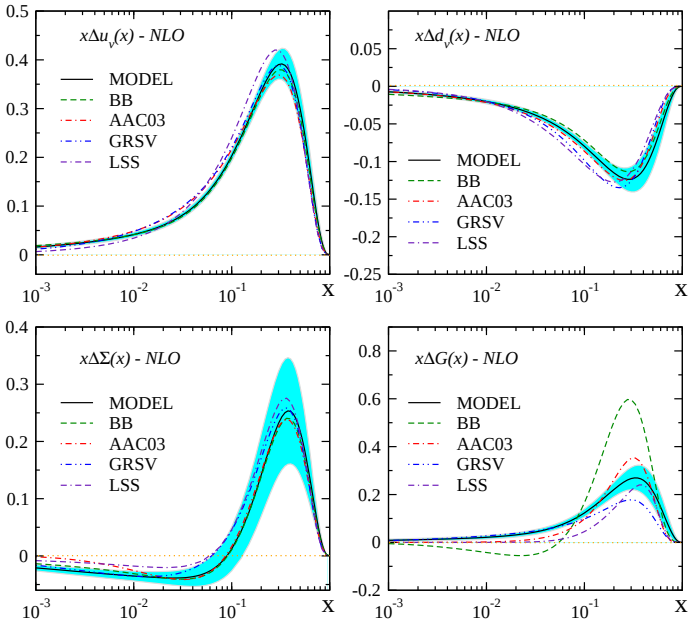

Figure 3: NLO polarized parton distributions at the input scale Q_0^2=1.0 GeV^2 compared to results obtained by BB model (dashed line) (ISET=3) [23], AAC (dashed-dotted line) (ISET=3) [24], GRSV (dashed-dotted dotted line) (ISET=1) [20] and LSS (dashed-dashed dotted line) (ISET=1) [27]

matic range shown for all Q^2. Also in this case the evolution moves the shape towards lower values of x and flattens the distribution.

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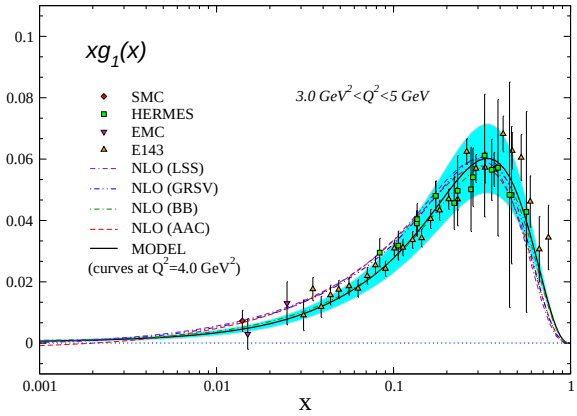

Figure 4: Polarized proton structure function xg_1^p measured in the interval $3.0\,\mathrm{GeV}^2 < Q^2 < 5.0\,\mathrm{GeV}^2$ as a function of x. Also shown are the QCD NLO curves at the same value of Q^2 obtained by LSS (dashed-dashed dotted line) (ISET=1) [27], GRSV (dashed-dotted dotted line) (ISET=1) [20], BB model (dashed-dotted line) (ISET=3) [23] and AAC (dashed line) (ISET=3) [24] for compar

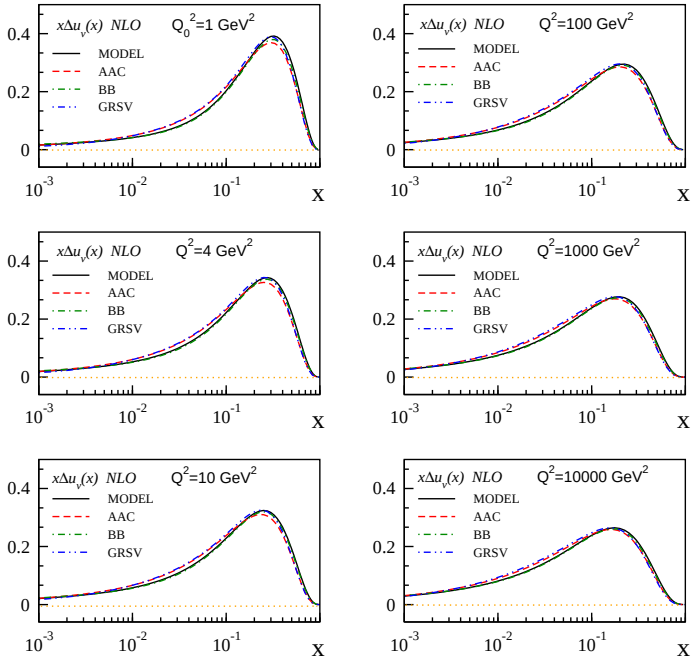

Figure 5: The Polarized parton distribution $x\Delta u_v$, evolved up to values of Q^2=10,000 GeV^2 as a function of x in the NLO approximation. The solid line is our model, dashed line is the AAC model (ISET=3) [24], dashed-dotted line is the BB model (ISET=3) [23] and dashed-dotted-dotted line is the GRSV model (ISET=1) [20].

By having polarized parton distributions, the first moments of the polarized parton distributions can be obtain. The first moments of the polarized parton densities in NLO

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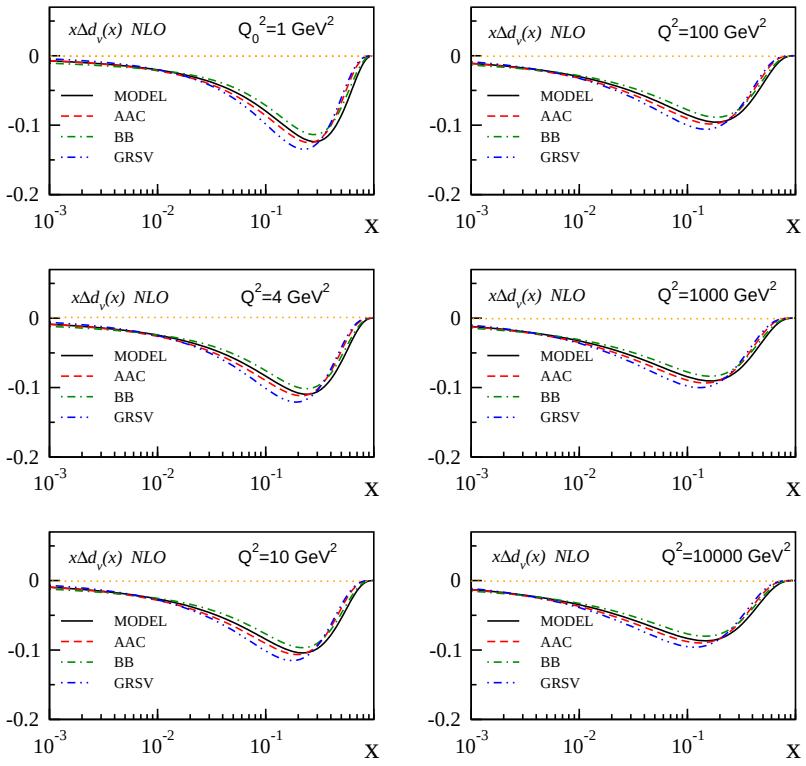

Figure 6: The Polarized parton distribution $x\Delta d_v$, evolved up to values of Q^2=10,000 GeV^2 as a function of x in the NLO approximation. The solid line is our model, dashed line is the AAC model (ISET=3) [24], dashed-dotted line is the BB model (ISET=3) [23] and dashed-dotted-dotted line is the GRSV model (ISET=1) [20].

Distribution	Model	ref. [25]	BB [23]	GRSV [20]	AAC [24]
Δu_v	0.9207	0.8769	0.926	0.9206	0.9278
Δd_v	–0.3391	–0.3313	–0.341	–0.3409	–0.3416
Δu	0.8390	0.8145	0.851	0.8593	0.8399
Δd	–0.4207	–0.3937	–0.415	–0.4043	–0.4295
$\Delta\overline{q}$	–0.0817	–0.0624	-0.074	–0.0625	–0.0879
ΔG	0.9325	0.8322	1.026	0.6828	0.8076

Table 3: Comparison of the first moments of the polarized parton densities in NLO in the $\overline{\text{MS}}$ scheme at $Q^2 = 4\,\text{GeV}^2$ for different sets of recent parton parameterizations. The second column (Model) contains the first moments which is obtained from the valon model and the Jacobi polynomials expansion method.

in the $\overline{\text{MS}}$ scheme at $Q^2 = 4\,\text{GeV}^2$ for different sets of recent parton parameterizations are presented in table 3.

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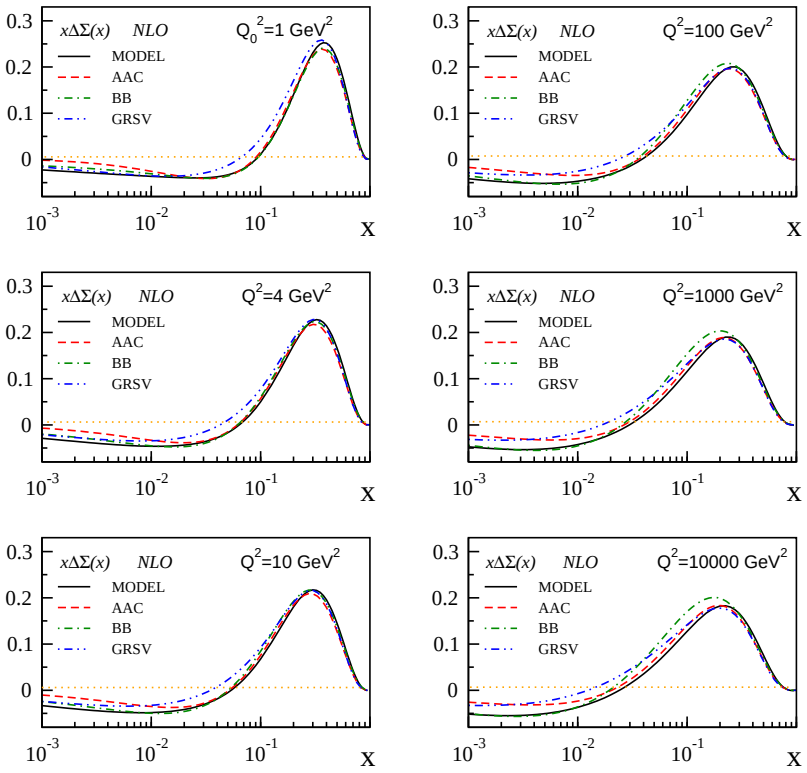

Figure 7: The Polarized parton distribution $x\Delta\Sigma$, evolved up to values of Q^2=10,000 GeV^2 as a function of x in the NLO approximation. The solid line is our model, dashed line is the AAC model (ISET=3) [24], dashed-dotted line is the BB model (ISET=3) [23] and dashed-dotted-dotted line is the GRSV model (ISET=1) [20].

$Q^2(GeV^2)$	Δu_v	Δd_v	$\Delta\Sigma$	$\Delta\overline{q}$	Δg	Γ_1^p
1	0.9260	-0.3410	0.0965	-0.0814	0.5850	0.1161
3	0.9215	-0.3393	0.0923	-0.0816	0.8651	0.1195
5	0.9202	-0.3389	0.0911	-0.0817	0.9837	0.1205
10	0.9189	-0.3384	0.0898	-0.0818	1.1383	0.1215

Table 4: The first moments of polarized parton distributions, Δu_v, Δd_v, $\Delta\Sigma$, $\Delta\overline{q}$, Δg and Γ_1^p in the NLO approximation for some value of Q^2.

We can also obtain the first moment of g_1^p (the Ellis-Jaffe sum rule) by

$$\Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x, Q^2) \; . \tag{6.1}$$

The results have also been given in table 4. The first moments of polarized parton distributions also shown for some value of Q^2.

In the framework of QCD the spin of the proton can be expressed in terms of the first moment of the total quark and gluon helicity distributions and their orbital angular

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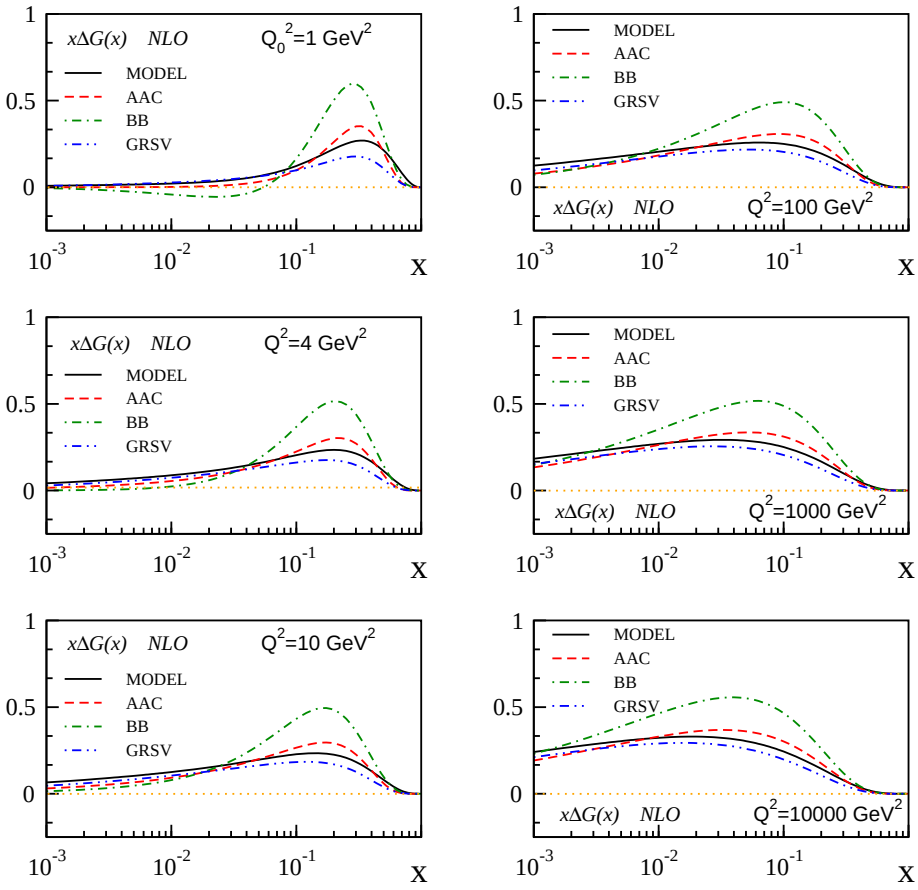

Figure 8: The Polarized parton distribution $x\Delta G$, evolved up to values of Q^2=10,000 GeV^2 as a function of x in the NLO approximation. The solid line is our model, dashed line is the AAC model (ISET=3) [24], dashed-dotted line is the BB model (ISET=3) [23] and dashed-dotted-dotted line is the GRSV model (ISET=1) [20].

momentum, i.e.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^p + \Delta g^p + L_z^p, \tag{6.2}$$

where L_z^p is the total orbital angular momentum of all the quarks and gluons. The contribution of addition of $\frac{1}{2}\Delta\Sigma$ and Δg for typical value of $Q^2 = 4\,\mathrm{GeV}^2$ is around 0.978 in our analysis. We can also compare this value in NLO with other recent analysis. The reported value from BB model [23] is 1.096, AAC model [24] is 0.837 and also GRSV model [20] is 0.785.

In the QCD analysis we parameterized the strong coupling constant α_s in terms of four massless flavors determining Λ_{QCD}. The LO and NLO results fitting the data, are

$$\begin{aligned} \Lambda_{\mathrm{QCD}}^{(4)\overline{\mathrm{MS}}} &= 201 \pm 110(stat)\ \mathrm{MeV}, \quad \texttt{LO}, \\ \Lambda_{\mathrm{QCD}}^{(4)\overline{\mathrm{MS}}} &= 245 \pm 58(stat)\ \mathrm{MeV}, \quad \texttt{NLO}, \end{aligned} \tag{6.3}$$

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These results can be expressed in terms of $\alpha_s(M_Z^2)$:

$$\begin{aligned}\alpha_s(M_Z^2) &= 0.1281 \pm 0.0094(stat) \ \text{MeV}, \quad \texttt{LO}, \\ \alpha_s(M_Z^2) &= 0.1141 \pm 0.0036(stat) \ \text{MeV}, \quad \texttt{NLO}. \end{aligned} \tag{6.4}$$

These values can be compared with results from other QCD analyzes of polarized inclusive deep-inelastic scattering data

$$\begin{aligned} \text{E154 [8]}: &\quad \alpha_s(M_Z^2) = 0.108 - 0.116\,, \\ \text{SMC [69]}: &\quad \alpha_s(M_Z^2) = 0.121 \pm 0.002(stat)\,, \\ \text{ABFR [10]}: &\quad \alpha_s(M_Z^2) = 0.120\, {}^{+0.004}_{-0.005}\,(\text{exp})\, {}^{+0.009}_{-0.006}\,(\text{theor})\,, \\ \text{BB [23]}: &\quad \alpha_s(M_Z^2) = 0.113 \pm 0.004(stat), \quad (\text{ISET=3})\,, \end{aligned} \tag{6.5}$$

and with the value of the current world average

$$\alpha_s(M_Z^2) = 0.118 \pm 0.002 \ \ [70]\ . \tag{6.6}$$

We hope our results of QCD analysis of structure functions in terms of Jacobi polynomials could be able to describe more complicated hadron structure functions. We also hope to be able to consider the symmetry breaking of polarized sea quarks by using the polarized structure function expansion in the Jacobi polynomials.

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